DESIGN OF A 207 FT. SPAN SPANDREL-BRACED TWO-HINGED ARCH

BY

R. L. STEVENS and W. TRINKAUS

Armour Institute of Technology

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Stevens, R. L.
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spandrel braced two-hinged

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[illegible]

The arch is a structure so arranged [illegible] which result from loads are not parallel to the [illegible] the loads, but usually not [illegible] the springings of the arch. In arches of [illegible], [illegible], [illegible] in the form of ribs, the arch [illegible] member or members [illegible] section, and to loads, [illegible] itself to [illegible], constituting [illegible], [illegible] [illegible] in resisting [illegible] or imposed loads [illegible] stalled [illegible] arches resting [illegible].

In the spandrel-braced [illegible] member [illegible] [illegible] which the same [illegible].

[illegible] in any other kind of structure, the supporting forces or reactions caused by the loads on the structure, must be found. [illegible] truss not restrained from moving laterally at the supports the reactions are parallel to the loading forces, usually vertical and are found from the simple conditions of static equilibrium.

With the arch, there is, in addition to these vertical reactions, a thrust upon the abutments so that the reactions are inclined and are the resultants of the same vertical reactions which exist on a beam similarly loaded, combined with those directly opposing the horizontal thrust.

In arches of three hinges, the horizontal thrust [illegible] found. The hinges provide three points [illegible] of the arch section to bending is zero. Since there are [illegible] components of the reactions to be found — the vertical reaction at each end and the horizontal thrust [illegible] for the bending moment on the arch in terms of the [illegible] reaction, at each hinge, [illegible] equal to zero; [illegible] these points the arch [illegible] bending moment; and [illegible] allow for the three [illegible].

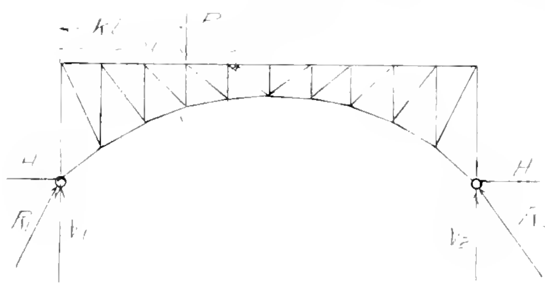
In arches of two hinges [illegible] which the bending [illegible] must be equal to zero, [illegible] the conditions of static [illegible] in the three [illegible] unknowns. The [illegible] be supplied [illegible] [illegible] in a [illegible].

The [illegible] expressed in [illegible] of the [illegible] and other [illegible]. [illegible] being subjected [illegible] [illegible] [illegible] as a function of the [illegible] of the arch [illegible], [illegible] [illegible] for both [illegible] and [illegible] of [illegible] [illegible] for the horizontal thrust, [illegible] to be [illegible]. [illegible] directly [illegible] the stresses [illegible]. In spandrel-braced arch [illegible] such a way that [illegible]. [illegible] so [illegible] of depth [illegible]

[illegible]

From this it is apparent [illegible]

[illegible]



[illegible] P [illegible] at [illegible] kl [illegible]

[illegible] R_1 and R_2 [illegible] V_1 [illegible] V_2 [illegible] H are [illegible]

[illegible] H [illegible]

[illegible]

By taking moments of the external forces about either [illegible], it is seen that

$$V_1 = P(1-k) \quad \text{and} \quad V_2 = Pk$$

which is the same as the reactions on a beam of equal span under the same load.

The value of H is the only part of the reactions now to be determined and in the following its value is deduced.

<u>Formula for Horizontal Thrust.</u>

The following notation will be used:

P = single vertical load on arch

kl = distance of load from left abutment

R_1 = reaction at left hinge

R_2 = " " right "

V_1 = vertical component of left reaction

V_2 = " " " right "

H = horizontal component of reactions

S'_n = stress which would exist in any member from the vertical components only of the reactions

T_n = stress which would exist in any member from a horizontal reaction of unity

S_n = actual stress in any member from load P

A_n = sectional area of any member

L_n = length of any member

E = modulus of elasticity of steel

δ_n = deformation of any member due to load P

Δ = horizontal deflection of arch which would take place under P if one end were free to move laterally.

There have come to our notice t[illegible] methods [illegible] ning an expression for the horizontal thrust. A[illegible] of the elastic deformation of the arch and simply [illegible] employment of differe[illegible] the result.

The first method [illegible] L[illegible] Forms [illegible] American Society of Civil Engineers, [illegible], [illegible]-Breslau, "Graphic Statics."

[illegible] principle [illegible] dis- tributed [illegible] of the load [illegible] will satisfy [illegible] rive, then the [illegible] will [illegible] will be a minimum. [illegible] known quantities [illegible] which are unknown [illegible].

The value [illegible] to the unknown forces equal [illegible] forces. From [illegible] principle [illegible] (From, "[illegible] Statics.")

I. The [illegible] of [illegible] of [illegible] force acting on a body [illegible] of the [illegible] to the first derivative [illegible] resistance produced in the body, [illegible].

II "The partial derivatives [illegible] to the statically indeterminate [illegible] that the [illegible]."

This principle [illegible] applied [illegible]:

The [illegible]

$$dW = \frac{1}{2} S \delta$$

From Hooke's Law [illegible],

$$\delta = \frac{SL}{AE}$$

$$\therefore dW = \frac{S^2 L}{2AE}$$

The [illegible],

$$W = \Sigma \frac{S^2 L}{2AE}$$

[illegible]

$$S = S' + HT$$

$$\therefore W = \Sigma \frac{(S' + HT)^2 L}{2AE}$$

From Castigliano's [illegible];

$$-\Delta l = \frac{dW}{dH}$$

Where Δl is [illegible])

[illegible] $\frac{dW}{dH}$. [illegible].

[illegible]

$$W = \Sigma \frac{(S'^2 + 2S'TH + T^2H^2)L}{2AE}$$

$$\therefore \frac{dW}{dH} = \Sigma \frac{(2S'T + 2T^2H)L}{2AE}$$

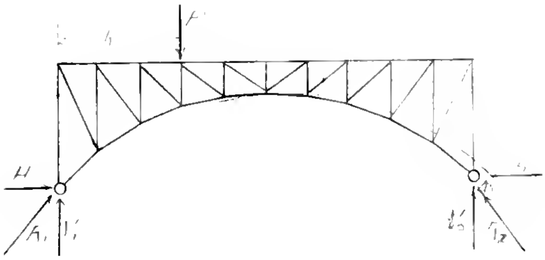
$$\therefore -\Delta l = \Sigma \frac{S'TL}{AE} + H \Sigma \frac{T^2L}{AE}$$

$$H = -\frac{\Sigma \frac{S'TL}{AE} + \Delta l}{\Sigma \frac{T^2L}{AE}}$$

If [illegible] $\Delta l = 0$)

$$\therefore H = -\frac{\Sigma \frac{S'TL}{AE}}{\Sigma \frac{T^2L}{AE}}$$

A [illegible] bridges. [illegible]



[illegible]

[illegible] P [illegible]

the length [illegible]

[illegible]

[illegible] just suffic[illegible]

we [illegible]

[illegible]

[illegible] horizontal force [illegible]

[illegible] hinge is [illegible]

[illegible]

[illegible] the first case, [illegible]

[illegible]

[illegible] P [illegible] Q [illegible]

[illegible] Q, T'

stress in any member caused [illegible] Q and δ the corresponding deformation in the member.

The external work done by Q in causing the deflection Δ' is

$$W = \frac{1}{2} Q \Delta'$$

and the internal work in any member is

$$dW = \frac{1}{2} T' \delta$$

The total internal work of deforming the truss is the sum of the work done on each member, or

$$W = \frac{1}{2} \Sigma T' \delta$$

Since the total [illegible] the external work

$$Q \Delta' = \Sigma T' \delta$$

P is the load [illegible] H, [illegible] stresses S', [illegible] λ, [illegible]. Δ [illegible] movement of hinge due to P. [illegible] $\frac{1}{2} H \Delta$, [illegible] work $= \frac{1}{2} \Sigma S' \lambda$

and

$$\lambda = \frac{S' l}{A E}$$

The deformation [illegible] the sum of [illegible].

Let Δ_1' [illegible] Q to the deformation [illegible].

$$\therefore \frac{1}{2} Q \Delta_1' = \frac{1}{2} T \delta$$

$$\therefore \Delta_1' = \frac{S}{H} \lambda = \frac{T}{Q} \delta$$

Likewise, let Δ_1 [illegible] load P [illegible].

$$\therefore \frac{1}{2} H \Delta_1 = \frac{1}{2} S' \lambda$$

$$\therefore \Delta_1 = \frac{S'}{H} \lambda$$

[illegible] $\frac{S'}{H} = \frac{T}{Q}$ since [illegible] the member divided [illegible] force [illegible] at [illegible], [illegible] causes the stress. (Stresses [illegible] directly [illegible] tional to the intensity [illegible] forces causing them.)

Substituting for $\frac{S'}{H}$ its equal, $\frac{T}{Q}$,

$$\Delta_1 = \frac{T}{Q}\lambda = \frac{T}{Q}\frac{S'L}{AE}$$

This is the portion of [illegible] of the hinge due [illegible] [illegible] the total deflection [illegible] sum of these portions [illegible],

$$\Delta = \Sigma\Delta_1 = \Sigma\frac{T}{Q}\frac{S'L}{AE}$$

Since the actual value of Q is [illegible], let its value [illegible] as unity, so that

$$\Delta = \Sigma\frac{S'TL}{AE}$$

This gives the deflection [illegible] load P.

We have now to derive another value of Δ [illegible] H in [illegible] back [illegible]. Let H be the force [illegible]. Δ [illegible] the [illegible]. The stress [illegible] HT [illegible] deformation, δ.

$\therefore$ External [illegible] $= \frac{1}{2}H\Delta$ [illegible] $\frac{1}{2}HT\delta$

$$\therefore \Delta = \Sigma T\delta$$

But

$$\delta = \frac{(HT)L}{AE}$$

$$\therefore \Delta = \Sigma\frac{T^2HL}{AE}$$

$$= H\Sigma\frac{T^2L}{AE}$$

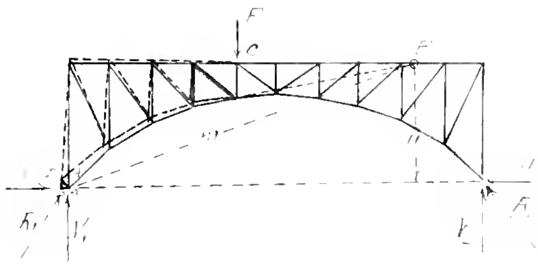
Equating the two values of Δ [illegible],

$$\sum \frac{STL}{AE} = H \sum \frac{T^2 L}{AE}$$

$$\therefore H = \frac{\sum \frac{STL}{AE}}{\sum \frac{T^2 L}{AE}}$$

[illegible] H [illegible]

[illegible]



[illegible]

[illegible] m [illegible] U [illegible] λ [illegible] d [illegible] AD.

$$\therefore \frac{d}{m} = \frac{\lambda}{V}$$

[illegible] Δ_1 [illegible] AE [illegible] y [illegible]

d y

m,

$$\therefore \frac{\Delta_1}{y} = \frac{d}{m} = \frac{\lambda}{V}$$

$$\therefore \Delta_1 = \frac{\lambda}{V} Y$$

I. HT H, S'

S

$$\therefore S = S' + HT$$

$$\lambda = \frac{SL}{AE}$$

$$= \frac{S'L}{AE} + \frac{HTL}{AE}$$

$$\therefore \Delta_1 = \frac{\lambda}{V} Y = \frac{S'L}{AE} \cdot \frac{Y}{V}$$

$$= \frac{S'L}{AE} \frac{Y}{V} + H \frac{TL}{AE} \frac{Y}{V}$$

$$\frac{Y}{V} = T$$

$$\therefore \Delta_1 = \frac{S'TL}{AE} + H \frac{T^2L}{AE}$$

$$\Delta = \Sigma \Delta_1 = \Sigma \frac{S'TL}{AE} + H \Sigma \frac{T^2L}{AE}$$

$$\therefore \Delta = \Sigma \frac{S'TL}{AE} + H \Sigma \frac{T^2L}{AE}$$

$$= \Sigma \frac{S'TL}{AE}$$

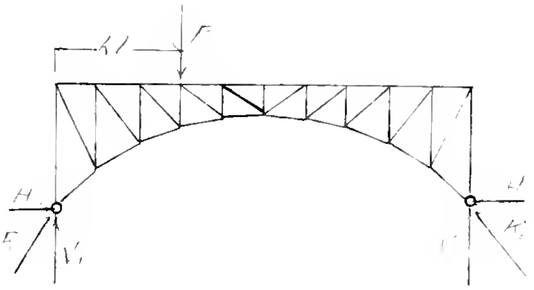
$$\therefore H = - \frac{\Sigma \frac{S'TL}{AE}}{\Sigma \frac{T^2L}{AE}}$$

$$\frac{S'TL}{AE}$$

$$\Sigma \frac{T^2L}{AE}$$

E

$$H = \frac{\sum \frac{S'TL}{A}}{\sum \frac{T^2L}{A}}$$



of

F

$$V_1 \frac{u}{v}$$

$$S' = V_2 \frac{u}{v}$$

$$S' = (V_1 + V_2)\frac{u}{v}$$
$$= F\frac{u}{v}$$

$$S' = \frac{1}{v}(V_1 u - P(u - kl))$$

$$S' = \frac{V_2 u}{v}$$

For the stresses in both members,

$$S' = (V_1 + V_2)\frac{u}{V} + \frac{1}{V}(Pl - Pu - V_1 l)$$
$$- P\frac{l}{V} - \frac{Pl}{V} + P\frac{kl}{V}$$
$$- Pk\frac{l}{V}$$

Since the truss is symmetrical, T, the stress due to horizontal thrust is the same in the symmetrical members.

Substituting for S' in the formula for H

$$H = \frac{\Sigma_0^{kl} P \frac{u}{V} \frac{TL}{A} + \Sigma_{kl}^{\frac{1}{2}l} P \frac{kl}{V} \frac{TL}{A}}{2\Sigma_0^{\frac{1}{2}l} \frac{T^2L}{A}}$$

the summation covering the members in one half the span.

$$\therefore H = P\frac{\Sigma_0^{kl} \frac{u}{V} \frac{TL}{A} + \Sigma_{kl}^{\frac{1}{2}l} \frac{kl}{V} \frac{TL}{A}}{2\Sigma_0^{\frac{1}{2}l} \frac{T^2L}{A}}$$

Let n be the number of the panel point from the left end at which the load P is acting. Let p be the panel length.

Then the distance of the load from the left end,

$$kl = np.$$

$$\therefore H = P\frac{\Sigma_0^{kl} \frac{u}{V} \frac{TL}{A} + n\Sigma \frac{p}{V} \frac{TL}{A}}{2\Sigma \frac{T^2L}{A}}$$

The stress in any member,

$$S = S' + HT$$

Let N be the number of panels in the truss.

$$\therefore l = Np$$

$$V_1 = P\frac{l - kl}{l} = P\frac{Np - np}{Np}$$

$$= P\frac{N-n}{N}$$

$$\therefore S' = V_1\frac{u}{V} = P\frac{N-n}{N}\frac{u}{V}$$

for members to the left of P

and

$$S' = \frac{V_1 u}{V} - \frac{P(u - np)}{V}$$

$$= P\frac{N-n}{N}\cdot\frac{u}{V} - P\,\frac{u-np}{V}$$

$$= P\left(\frac{N-n}{N}\cdot\frac{u}{V} - \frac{u-np}{V}\right)$$

for members between P and the middle of the truss.

Substituting these values of S' and H in the formula for S

$$S = P\left[\frac{N-n}{N}\cdot\frac{u}{V} - \frac{u-np}{V} + \left(\frac{\Sigma_0^{kl}\frac{u}{V}\frac{TL}{A} + n\,\Sigma_{kl}^{l}\frac{p}{V}\frac{TL}{A}}{\Sigma_0^{l}\frac{T^2L}{A}}\right)T\right]$$

which gives the stress in any member due to load P at panel point n. The second term is dropped, however, except for members between P and the middle of the truss.

The Design.

The bridge selected to be designed according to [illegible] is of the same general dimensions as the one designed and built in 1902 by the Chicago, Milwaukee and St. Paul Railroad at Iron Mountain, Michigan, as a three-hinged steel arch. The span is 207' and the depth 52'. It is a single track deck structure with trusses spaced [illegible].

The bridge crosses the Menominee River and at that point the banks consist of solid granite so that the situation is ideal for an arch span.

For this design the crown depth was assumed as [illegible] feet, the curve of the lower chord as a parabola, and the span was divided into ten panels of [illegible] each. The same loadings, unit stresses and specifications were used so that the two designs serve in a measure as a basis for comparison of the two classes of arches. The outline of one-half of the truss with the lengths and areas of the members is given in Plate 1.

The live load is that known as Cooper's "Class E-50 Loading" except that the uniform load following the two locomotives was assumed as 7000 pounds per foot of track instead of 5000 pounds to allow for the excessive weight of ore trains. The intensity of the uniform load was so great that it was used instead of locomotive concentrations in finding the stresses in the trusses. For the floor system the moments and shears were greater for the concentrated loads than for the uniform load. The length of the locomotive (without tender) wheel base being nearly equal to the panel length, the difference between its weight and that of an

equal length of uniform load was taken and an "excess" load [illegible] was applied to two such alternate panel points as would produce the maximum stress in each member. The details of the loading are given in table 1.

The fact that the bridge is anchored only at the abutment hinges, while the uppermost portion of the trusses and connections and the live loads exposed to wind action are at a considerable distance above the abutments, gives rise to large overturning moments which act to produce loads on the truss, acting downward on the leeward side and upward on the windward side.

The distribution of the wind stresses among the various systems of bracing is indeterminate in this kind of a structure, but by making the upper lateral bracing of nominal dimensions we have considered that the loads applied at the upper panel points are carried down through the sway bracing to the lower panel points and from there through the lower system of lateral bracing to the abutments. This seemed to be the most direct way of transferring the wind loads to the abutments and was taken as the most probable.

The wind on the train was assumed to act [illegible] feet above the middle of the upper chord and was treated as live load. This live wind load and that applied at the upper chord produces an overturning moment about the corresponding lower chord panel points. Since the lower chord panel points are not in the same horizontal plane, the load at each panel point produces an overturning moment about the next panel point toward the abutment. The loads are given in Table 1. In addition to the vertical

loads on the truss [illegible] overturning, the horizontal wind loads [illegible]ing on the lower chord and transfered to it by the sway frames [illegible]duce stresses in the lower lateral system. A graphical determination of the stresses in the lower lateral system is given on Pl[illegible]e II and in Table II is given the composition of the lower lateral system.

The design of the intermediate sway [illegible] is given [illegible] III. The stresses in the end sway bracing and in the [illegible] beam are given, graphically, in Plate IV [illegible] IV. The design of the floor system is given [illegible]

As a preliminary to the computation [illegible] of constants for the members of the truss was computed. [illegible] independent of the loads on the truss and are given in [illegible] V.

The stresses in each member due to loads of unit[illegible] ious panel points, considering only the effect of the vertical [illegible]actions, were next computed. This corresponds [illegible] the terms

$$\frac{N-n}{N}\,\frac{u}{V} - \frac{u-np}{V}$$

The former is given in Table VI and the latter in Table VII for the members and loads to which it applies. They are combined i[illegible] Table VIII.

The term $\sum \frac{A}{V} TL + n \sum \frac{p}{V} TL$

was next computed. Since in this first trial we have [illegible]othing to determine the areas of the sections, they are assumed for this purpose to be all equal so that the term cancels out of the expression for H. The values for this expression are given in Table IX, the quantities for each class of members above the

heavy lines being computed from the first term and those below from the second. Since for panel loads beyond 4 there are no members to the right of the load, then for all succeeding panel loads only the first term applies. The summations are obtained by adding all the quantities for one panel load, all the S'ns being minus as S'ns of opposite sign to T. The summations for each panel load are divided by the quantity,

$$L' \sum_0^{2L} T^2 L$$

from Table V, and the result is the value of

$$\frac{\sum \frac{M}{V} TL + \sum n \frac{P}{V} TL}{\sum T^2 L}$$

for each unit panel load in the expression for H.

In Table X these values have been multiplied by the value of T for each member.

In Table XI these stresses from horizontal reactions are combined with those from vertical reactions in Table VIII and the results are the actual stresses in the members from unit panel loads.

Since for dead load, all panel loads have to be considered as acting at all times and as the same load is concentrated at the corresponding panel points from the two ends, some labor is saved by combining the corresponding unit stresses before multiplying them by the panel loads. These are given in Table XII. Since the live and "excess" panel loads are all equal, the unit stresses which will produce to largest values, plus and minus, are combined in Table XII. In Table XIII are given the stresses due to dead panel loads, being the values in Table XII multiplied by the panel loads. These are combined and are given in the

column for Dead Load in Table XV. In Table XIV are given the stresses due to wind loads, and under Wind Load in Table XV the sums of the stresses of the same sign are given. For the live loads the maximum possible stresses of [illegible] kind will take place when every panel point which gives stress of that kind is loaded and when all panel points which have stress of the opposite kind are unloaded. In the columns for live load and [illegible] load the quantities in the corresponding columns for unit stresses in Table XIII are multiplied by the panel loads. The specified unit stresses [illegible] sectional area for live load stresses shall be twice that provided for equal dead load stresses. For this reason the [illegible] stresses in the last two columns are the sum of the live stresses and half the dead load stresses, and the live load [illegible] stresses used in designing. In Table XVI is given the preliminary design of the members from which areas were obtained [illegible] used in the second trial [illegible]. At this point it was thought that to provide [illegible] in this preliminary the stresses should be increased [illegible]. The results of the second trial [illegible] necessary. Since the [illegible] have a [illegible] proportion of the total stresses, [illegible] specifications [illegible] 0 [illegible] used.

From the areas figured, a new analysis of dead load was made computed and was used instead of the original dead load in the [illegible] design. The weights were figured from the areas [illegible].

to provide for the additional weight of construction an increase of 10 % was made. The dead load panel weights are given in Table XVII.

In Table XVIII are given the computations of several constants in the formulas, including the camber of the truss.

The temperature stresses were figured from a modification of the formula for horizontal thrust. In the derivation of the formula for horizontal thrust by the method of least work it was found that

$$H = [illegible]$$

where Δl is the change in span.
If for Δl is put the value of the change of span which would take place from the change in length of members from change in temperature if the truss were unrestrained, the resulting value of H would be the thrust upon the abutments caused by that tendency to change of span.

$$[illegible]$$

where [illegible] is the coefficient of linear expansion per degree Fahrenheit for steel, t the change in temperature degrees and l the normal length of span. The term

$$[illegible]$$

is zero since for horizontal reaction only [illegible] is zero.

The stress in any member due to temperature will then be the horizontal thrust due to that change multiplied

The following values for the quantities were [illegible].

$$E = [illegible]000 \text{ kips}$$

$$\epsilon = .00000[illegible]$$

$$l = [illegible]$$

$$t = +[illegible]0 \ldots - 0$$

$$\therefore S_t = \frac{[illegible]000 \times .0000[illegible]}{[illegible]} \frac{T}{\Sigma T^2 \frac{l}{A}}$$

$$= [illegible] \quad \frac{T}{\Sigma T^2 \frac{l}{A}}$$

From Table XVIII

$$\Sigma T^2 \frac{l}{A} = 24.[illegible]5$$

so that $\qquad S_t = [illegible].14 \; T$

The values of S_t for each member is given in the last column in Table XVIII.

Tables XIX to XXVI are similar [illegible] and are self explanatory.

In Table XXVII is given [illegible] designs of [illegible] sections as finally adopted [illegible] of the arch.

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